

## Rethinking Stakeholders' Harmony: Plan 2014? Plan 2024!

### Summary

The water level management of the Great Lakes has an important impact on the ecology, economy, society and other aspects of North America. Adequately balancing the desires of the various stakeholders on the Great Lakes is not an easy task. We established a set of management and control strategies that seek to better benefit all stakeholders.

In **TASK 1**: We established the weighted **digraph** and **Water Balance Equation** based on the Great Lakes for communicating flow relationships between different waters. We first analyzed the relationship between these desires and the three variables of water level  $h_i(t)$ , outflow  $Q_{out}(t, i)$ , and water speed  $v_i(t)$ . Using **TOPSIS evaluation method** and **mechanism analysis method**, we established the standardized interest quantification indicators of six stakeholders. We reduced outflow and water speed to a function of water level through **surface integration** and **LSTM**, respectively, and carried out a **multi-objective optimization** with the water level of each water area as the decision variable and the weighted sum of each indicator as the objective function. Ultimately, the time-varying optimal water level for each water area was obtained to be roughly between 69.274% and 87.342% of the highest water level of the year.

In **TASK 2**: First we established the **Cylinder Equivalent Model of Water Capacity**, which established a bijection between water volume and water level and defines the optimal capacity. And then, we developed a system of **differential equations model** concerning water storage to solve for its time-varying function. Based on the structure of the **plan 2014** algorithm and **J-limit**, we introduce a flow factor  $\tau$  containing an undefined parameter set  $(C_1, C_9)$  to solve the problem of its poor performance in the face of sudden changes in water flow. Finally, by solving the ODE under parameter optimization, the optimal parameter set of  $(C_1, C_9) = (3.4, 2.7)$  is solved, and the control algorithm for maintaining the optimal water level in the Great Lakes, Plan 2024, is finally obtained.

In **TASK 3**: To assess the sensitivity of  $Q_{out}$  from two dams under Plan 2024, we analyzed **sensitivity coefficients**  $S(x, para)$  with parameters  $(C_1, C_9)$ . The analysis showed that  $Q_{out}(t, i)$  adjustments through these parameters are minimally sensitive. A retrospective analysis using 2017 data, compared to Plan 2014, revealed that Plan 2024 offered improved benefits for stakeholders, exceeding actual water levels of that year.

In **TASK 4**: To evaluate the sensitivity of Plan 2024 to environmental conditions, we defined the relationships among **precipitation**, **snowpack**, **ice jams**, and water holding capacity as perturbation terms within a system of differential equations. Subsequently, by calculating the time-varying function of water capacity and its sensitivity coefficients to these perturbations, it was found that the approximate optimal water capacity is insensitive to the perturbations.

In **TASK 5**: We focused on the water level management of Lake Ontario and utilized **multiple linear regression** to establish the numerical relationship between water levels and stakeholders' indicators. Extensive analysis was conducted on the impact of various stakeholders on the optimal water level.

In summary, our model is designed to maximize benefits for all stakeholders.

**Keywords:** Plan 2024, TOPSIS, Water Balance Equation, Multi-Objective Optimization

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# 1 Introduction

## 1.1 Background

The Great Lakes and their drainage basins are vast and characterized by varying climatic and weather conditions. Managing and modeling the two primary control mechanisms in the flow of water in the Great Lakes system, Compensating Works and Moses-Saunders Dam, taking into account relevant natural and policy factors, can help to ensure the interests of all stakeholders.

## 1.2 Our Work

The work we have done in this paper is mainly shown in the following Figure 1.

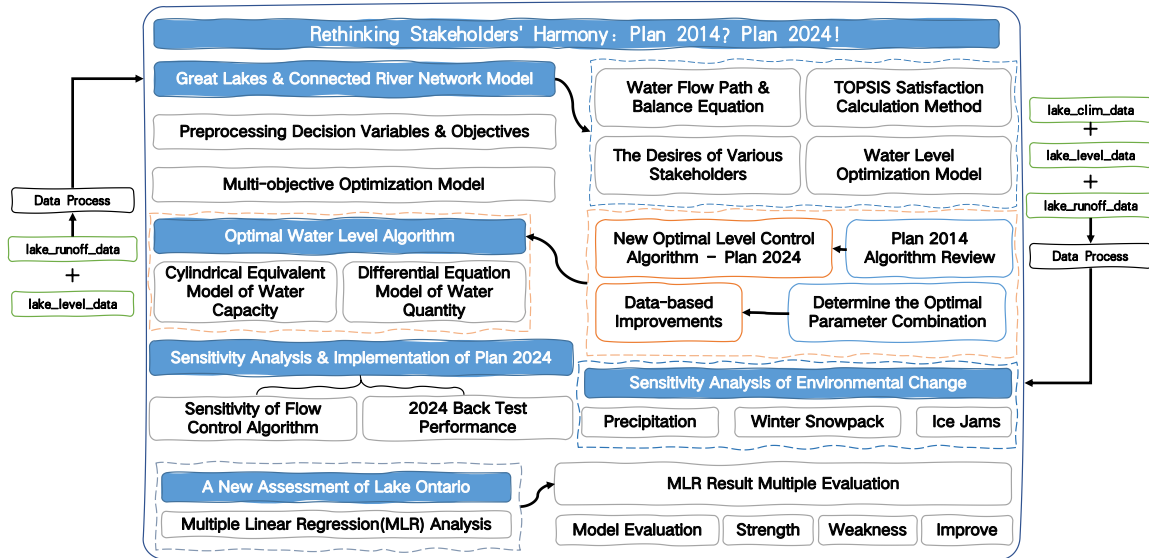


Figure 1: Our Work

## 2 Problem Statement and Analysis

In order to achieve the goal mentioned in section 1.1, we need to accomplish the following requirements based on a network model consisting of the Great Lakes system.

- 1. Determination of optimal water level:** Determining optimal water levels for the Great Lakes requires not only considering changes in water levels at various times of the year in the Great Lakes, but also taking into account the needs and costs of various stakeholders. We therefore create different indicators for measuring the interests of different stakeholders and transform the problem into a form of multi-objective optimization to solve for the optimal water level.
- 2. Establishment of control algorithm for the optimal water level maintenance:** Design algorithms based on water flow data to dynamically adjust and maintain optimal water

levels. Consider Plan 2014 based control algorithms and add more control factors to respond to a variety of complex factors.

3. **Sensitivity analysis of control algorithms:** Analyze the sensitivity of control algorithms for the outflow of the two control dams and evaluate algorithm performance based on 2017 data.
4. **Impact of environmental changes on the control algorithm:** Analyze the sensitivity of control algorithms for the variation of environmental conditions.
5. **Specific analysis of Lake Ontario Assessment:** A Lake Ontario-specific analysis discusses its water level management in the context of regional characteristics.

### 3 Preparation of the Models

#### 3.1 Assumptions

1. For a specific water area, take a flow section of the water area, at the same time, the flow speed of each position of the section is the same.
2. Since the data of Lake Huron and Lake Mississippi are merged, we regard them as the same lake for modeling.
3. The water level change near the water surface is not significant compared to the depth of the water, so the water boundary near the water surface can be regarded as a regular vertical surface. Therefore, given the difference of water level change, the volume change can be calculated according to the surface area of the water surface.
4. All the data that can be found are almost reliable, and the impact of their small errors on the accuracy of the model is negligible.
5. The constant coefficients of all quantitative indicators are available.

Table 1: Notations

| Symbol          | Definition                                                          |
|-----------------|---------------------------------------------------------------------|
| $t$             | The time of year                                                    |
| $i$             | A categorical variable refers to a specific water area numbered $i$ |
| $Q(t, i)$       | The water flow of water area $i$ at time $t$                        |
| $Q_{in}(t, i)$  | The inflow of water area $i$ at time $t$                            |
| $Q_{out}(t, i)$ | The outflow of water area $i$ at time $t$                           |
| $v_i(t)$        | The water speed of water area $i$ at time $t$                       |
| $h_i(t)$        | The water level of water area $i$ at time $t$                       |
| $h_i^*(t)$      | The optimal water level of water area $i$ at time $t$               |
| $V_i(t)$        | The water capacity(volume) of water area $i$ at time $t$            |
| $V_i^*(t)$      | The optimal water capacity(volume) of water area $i$ at time $t$    |
| $g$             | The coefficient of gravity                                          |

Note :  $Q_{out}(t, i)$  and  $Q_{in}(t, i)$  are concrete forms of  $Q(t, i)$ .

### 3.2 Notations

The primary notations used in this paper are listed in Table 1.

## 4 Task 1: Optimal Water Level based on the Desires of Various Stakeholders

### 4.1 The Network Model of the Great Lakes Region and the Connected Rivers

The Five Great Lakes network model is a weighted digraph, defined as  $G = (V, E)$ , where  $V$  is the vertex set and  $E$  is the arc set.

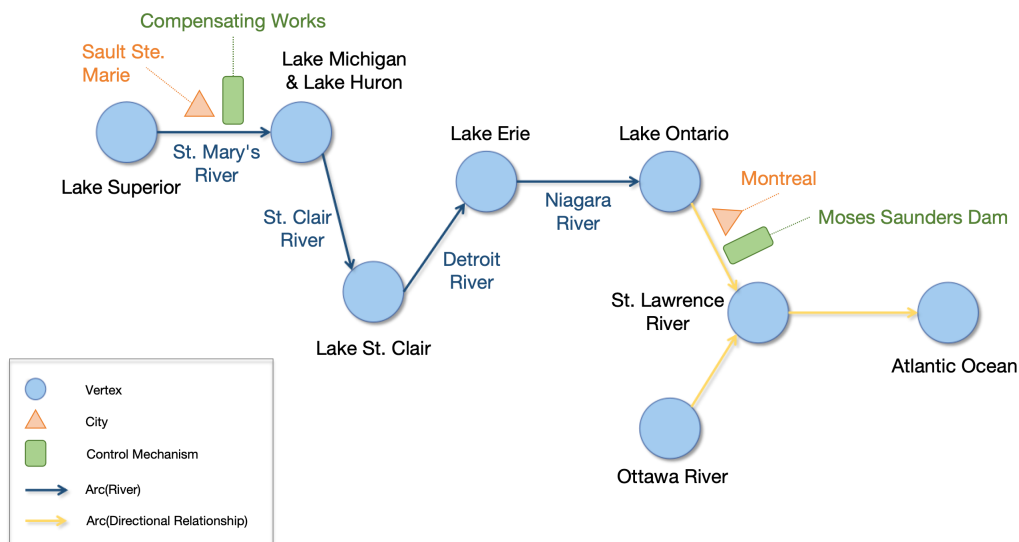


Figure 2: Water Flow Network of the Five Great Lakes

- **Vertex:** The vertex is often a lake in the Great Lakes region ( not just these five great lakes ), but it may also be a river or the Atlantic Ocean. Based on the geographic relationship of lakes and rivers, we chose Lake Superior, Lake Michigan & Lake Huron, Lake St. Clair, Lake Erie, Lake Ontario, St. Lawrence River, Ottawa River and Atlantic Ocean as our vertices.

- **Arc:** The water flows from west to east into the Atlantic Ocean from Lake Superior. Naturally, we choose St. Mary's River, St. Clair River, Detroit River, Niagara River and three other edges with flow relations to connect the corresponding vertices. The direction of the arc in the arc set represents the direction of the water flow. If the head of the arc is not a 'lake' vertex, then this arc only means a directional relationship of the water flow. Otherwise, this arc refers to a river.

- **Weight:** The capacity of water and has no fixed value.

In addition, Compensating Works of the Soo Locks at Sault Ste. Marie and Moses-Saunders Dam at Cornwall are two of the major control mechanisms in the flows of the Great Lakes system. Sault Ste. Marie and Montreal are important cities that are located

near both flow basins and control mechanisms. We will consider them so that we can more clearly consider the impact of various stakeholders and complex factors on the research question.

## 4.2 Water Flow Path and Water Balance Equation

**Definition: Water Flow Path  $\mathcal{P}_i$**   $\mathcal{P}_i$  is an ordered sequence of positive natural number, in which the value of the element refers to the water corresponding to the order in the flow path, which is expressed as :

$$\mathcal{P}_i = \{1, 2, 3, \dots, n_i\}$$

where  $i$  refers to a particular area of water on the flow path  $\mathcal{P}_i$ , and the  $n_i$  here means that the water flow finally stops at the  $i$  water area. Obviously, the longest path in this digraph is the ordered sequence  $\mathcal{P} = \{1, 2, 3, \dots, n\}$ , which is the flow from Lake Superior to Atlantic. It flows through a total of  $n$  waters.

There exists  $i - 1$  water area, whose outflow is  $Q_{\text{out}}(t, i - 1)$  and inflow is  $Q_{\text{in}}(t, i - 1)$ . Then we have the following equations, which is called the *Water Balance Equation*:

$$\begin{cases} V_i(t) = V_i(t - 1) + [Q_{\text{in}}(t, i) - Q_{\text{out}}(t, i) - Q_{\text{quote}}(t, i)] \cdot \Delta t \\ Q_{\text{in}}(t, i) = \sum_{j \in N^-(i)} Q_{\text{out}}(t, j) \end{cases} \quad (1)$$

where  $N^-(i)$  represents the source of water upstream of  $i$  water area. In the water flow path  $\mathcal{P}_1$ , it is clear that there is

$$N^-(i) = i - 1.$$

$V(t, i)$  represents the water capacity of  $i$  at moment  $t$ .  $Q_{\text{quote}}(t, i)$  represents the flow of water consumed by  $i$  water area at moment  $t$  by power generation, human activities, etc.

## 4.3 TOPSIS Calculation Method of Satisfaction Degree

In the latter part, three satisfaction indicators will be calculated based on the TOPSIS algorithm. We make a unified methodological discussion here, The following is the process of TOPSIS calculation method.

**Step 1.** Create a matrix consisting of groups of data sets observed at  $N$  different times and  $J$  criteria. The corresponding criteria observations form the matrix  $A$ :

$$A = (a_{tj})_{N \times J} (t = 1, \dots, N \text{ and } j = 1, \dots, J)$$

For different stakeholders, the evaluation criteria is different.

**Step 2.** Each element of matrix  $A$  is processed with uniformization (the higher its value the better the metric), then use the 'range change method' to standardize the metric, that is :

$$b_{tj} = \frac{a_{tj} - \min_{1 \leq t \leq N} a_{tj}}{\max_{1 \leq t \leq N} a_{tj} - \min_{1 \leq t \leq N} a_{tj}} (1 \leq t \leq N, 1 \leq j \leq J). \quad (2)$$

Hence, the evaluation matrix  $B$  is obtained,  $B = (b_{tj})_{N \times J} = (\vec{b}_t)_{N \times 1}$ .

**Step 3.** Determine the positive ideal solution  $\mathbf{C}^+$  and the negative ideal solution  $\mathbf{C}^-$ .

$$\begin{cases} \mathbf{C}^+ = (c_1^+, \dots, c_J^+) \\ \mathbf{C}^- = (c_1^-, \dots, c_J^-) \end{cases} \quad \text{where } c_j^+ = \max_{1 \leq t \leq N} b_{tj}, c_j^- = \min_{1 \leq t \leq N} b_{tj}, \quad j = 1, 2, \dots, J.$$

**Step 4.** Calculate the distance to the positive ideal solution and to the negative ideal solution for each alternative.

$$\begin{cases} s_t^+ = \|\vec{b}_t - \mathbf{c}^+\|_2 \\ s_t^- = \|\vec{b}_t - \mathbf{c}^-\|_2 \end{cases}$$

**Step 5.** Calculate the relative similarity of each alternative to the ideal solution.

$$f_t = \frac{s_t^-}{s_t^+ + s_t^-} \quad (3)$$

$f_t$  is a maximum index, which can be given meaning according to the actual situation.

## 4.4 Analysis and Quantification of the Desires of Various Stakeholders

### 4.4.1 Information on Various Stakeholders

Determining optimal water levels for the Great Lakes requires a balanced consideration of the interests of various stakeholders. Based on the statements in the addendum, there are six main categories of stakeholders who are disputing the control algorithm in resolving the Lake Ontario sub-problem. According to the Plan 2014[3], we have analyzed and summarized the interests of the six categories of stakeholders in the following Table 2.

Table 2: Analysis of Stakeholders' Desires

| Stakeholders                      | Water Level       | Current          |
|-----------------------------------|-------------------|------------------|
| Municipal&Industrial Water supply | /                 | /                |
| Commercial Navigation             | high              | no current       |
| Hydropower                        | high              | large water flow |
| Coastal Development               | low               | no current       |
| Ecosystems                        | seasonal high/low | /                |
| Recreational Boating              | mid               | no current       |

### 4.4.2 Indicators of Benefits for Stakeholders

In the following discussion, we specify the number of days in a year as  $Y$ . We establish a series of indicators related to these stakeholders as follows.

#### • Average Daily Benefits of Companies responsible for Municipal and Industrial Water Supply $\xi$

In the water balance equation(1),  $Q_{quote}(t, i)$  refers to the way of water use in this part. The companies responsible for municipal and industrial water supply, draws the water



Figure 3: Stakeholders &amp; Indicators

out of the water area and provides it to the residents who need domestic water or industrial production, and thus makes a profit. Thus,  $\xi$  can be expressed as:

$$\xi = \sum_{i=1}^n \sum_{t=1}^N \gamma Q_{quote}(t, i) \Delta t / Y \quad (4)$$

where  $\gamma$  represents the benefit per unit of water used.

Indicator  $\xi$  measures the annual average benefits of Municipal and Industrial for all waters from Lake Superior to the Atlantic Coast.

• **Average Daily Transportation Benefits of Commercial Shipping Company  $\Phi$**

**Definition: Regional Transportation Benefits  $\lambda_i(t)$**

$$\begin{cases} \lambda_i(t) = \frac{W_s(t, i)}{T} \\ W_s(t, i) = \beta G_s(t, i) \\ G_s(t, i) = \rho_w S_{avg} g [h_i(t) - \inf_t h_i(t)] \\ T = \frac{D_i}{v_s - v_i(t)} \end{cases}$$

$\beta$  is the benefit per unit of cargo.  $\rho_w$  is the current density.  $S_{avg}$  is the average draft bottom area of the freighter.  $v_s$  is the ship's speed.  $v_i(t)$  is the speed of the water in area  $i$  at moment  $t$ .  $D_i$  is the mileage of the ship's transportation in water area  $i$ , which can be equated with length. And we end up with the following equation,

$$\lambda_i(t) = \frac{\beta \rho_w S_{avg} g}{D_i} [h_i(t) - \inf_t h_i(t)] [v_s - v_i(t)]. \quad (5)$$

Summing and averaging the transportation benefits for each region, we obtain average daily transportation benefits of commercial navigation company, expressed as

$$\Phi = \frac{\sum_{t=1}^N \sum_{i=1}^n \lambda_i(t)}{Y}. \quad (6)$$

The indicator  $\Phi$  measures the relationship between the average benefits of a shipping company's single cargo transportation and various factors, including water level and water speed.

• **Average Daily Benefits of Hydropower Generation Company  $W_e$**

We consider the benefits generated by the power generation of the hydropower company, that is, the revenue depends on the amount of power generation. Because the electric energy produced by the hydropower station is converted from the water energy, we use the water energy calculation formula[8] of the hydropower station to express the unit time power generation  $W_t$  of a power plant as follows.

$$W_t = E_e \Delta t = g \eta Q_{out}(t, i) \cdot (h_i(t) - h_{i+1}(t)) \Delta t \quad (7)$$

where  $\eta$  is the a parameter related to the scale of hydropower station, and  $E_e$  is the output of a hydropower station. Taking into account the actual power generation facilities of St. Ste. Marie Works and the Moses-Saunders Dam, we express  $W_e$  as:

$$W_e = num \cdot \alpha \sum_{t=1}^N W_t / Y \quad (8)$$

where  $\alpha$  is the benefit brought by unit electricity price, and  $num$  is the number of hydropower stations of the St. Ste. Marie Works and the Moses-Saunders Dam.

• **Average Daily Satisfaction of Coastal Property Owners  $F_{CD}$**

For coastal property owners, we selected  $h(t, i)$  water level and  $V_w(t, i)$  water velocity as evaluation criteria. Based on the TOPSIS calculation method, we sum and average the  $f_t$  obtained from equation(3) to obtain average daily satisfaction  $F_{CD}$  for coastal property owners.

$$F_{CD} = \frac{\sum_{i=1}^n \sum_{t=1}^T f_t(i)}{Y} \quad (9)$$

• **Average Daily Satisfaction of Environmentalists  $F_E$**

For environmentalists, their goal is to maintain the habitat for species to thrive and clean out static bays and. In spring and summer, they want the water level to rise, because in this way the nitrogen and phosphorus content in the water will rise, and many organisms such as algae can multiply quickly. In autumn and winter, they hope that the water level will drop, so that the sediments that adsorb nitrogen and phosphorus will be removed and the environment will be improved[7].

Hence, we consider using the concentration of element phosphorus  $C_P(t, i)$  and the concentration of element nitrogen  $C_N(t, i)$  as evaluation criteria. Similarly, we obtain average daily satisfaction  $F_E$  for environmentalists.

$$F_E = \frac{\sum_{i=1}^n \sum_{t=1}^T f_t(i)}{Y} \quad (10)$$

• **Average Daily Satisfaction of Recreational Boaters and Sport Fishing Industry  $F_{RB}$**

For recreational boaters and sport fishing industry, we also selected  $h(t, i)$  water level and  $V_w(t, i)$  water velocity as evaluation criteria. The corresponding satisfaction indicator is  $F_{RB}$ .

$$F_{RB} = \frac{\sum_{i=1}^n \sum_{t=1}^T f_t(i)}{Y} \quad (11)$$

## 4.5 Multi-objective Optimization Model of Water Level

### 4.5.1 Preprocessing of Decision Variables and Objectives

We have constructed the evaluation indicator system of each stakeholder, but the numerical scales of them in this system are different. We first standardize each indicator in order to measure them uniformly.

After standardization, we set the symbols of these standardized indicators as follows:

$$\xi' \ W_e' \ F_{CD}' \ F_{RB}' \ F_E' \ \Phi'$$

It is worth noting that the variables related to water flow in the quantitative indicators of various stakeholders are not only the water level height, but also the water speed  $v_i(t)$  and water flow  $Q(t, i)$ . However, if only the optimal water level is required, these variables need to be unified, so we find the equivalent relationship between them.

- The equivalent relation of  $v_i(t)$  and  $Q(t, i)$

The basic definition of water flow is: the volume of water flowing through the effective section in unit time. consider an infinitesimal  $ds$  at somewhere on the cross section  $A$ , and the unit time  $\Delta t$ , then:

$$\begin{cases} \Delta V_i(t) = \int_t^{t+\Delta t} \int_A v_i(t) ds dt \\ Q(t, i) = \lim_{\Delta t \rightarrow 0} \frac{\Delta V_i(t)}{S(A)} \end{cases} \quad (12)$$

where  $S(A)$  is the area of effective cross section of cross section  $A$ ,  $\Delta V_i(t)$  is the increment of water capacity of the whole water area. From the basic assumptions, the following integral equation is obtained :

$$\lim_{\Delta t \rightarrow 0} \int_t^{t+\Delta t} \int_A v_i(t) ds dt = S(A) v_i(t) \quad (13)$$

Combining (12) and (13), at a certain moment, there is the following equivalence relation:

$$Q(t, i) = v_i(t) \quad (14)$$

- The equivalent relation of  $h_i(t)$  and  $Q(t, i)$

$h_i(t)$  and  $Q(t, i)$  over time constitute the corresponding time series, and there is a non-linear relationship between the two sets of series. The influence of the relationship between  $h_i(t)$  and  $Q(t, i)$  of various complex factors exists regionally, that is, under different conditions in different regions, the relationship between  $h_i(t)$  and  $Q(t, i)$  of the equation in the value of the parameters often differ.

Long Short-Term Memory Network (LSTM) is a deep learning model commonly used to process time series data. It can effectively capture long-term dependencies and selectively store and transmit information through gate mechanisms and memory cells. Therefore, we used LSTM model to regress the data and analyze the relationship between  $h_i(t)$  and  $Q(t, i)$  in the Great Lakes system from the results. The model hyperparameters are configured as shown in the following Table 3.

Table 3: LSTM Model Hyperparameter Configuration

| Hyperparameter | Sequence Length | Batch Size   | Learning Rate | Epochs |
|----------------|-----------------|--------------|---------------|--------|
| Value          | 12              | 12           | 0.001         | 100    |
| Hyperparameter | Optimizer       | Hidden Layer | Hidden Size   | /      |
| Value          | Adam            | 1            | 50            | /      |

#### 4.5.2 Establishment of Multi-objective Optimization Model

In fact, different stakeholders are not necessarily equally important to us. For example, for river junctions, the satisfaction of riverside dock managers and residents is clearly more important than the benefits of shipping companies here, because high water levels are more urgent for the former 's life and life threats. Therefore, we will assign different weights to different goals. i.e.

$$0 \leq w_k \leq 1 \quad k = 1, 2, \dots, 6 \quad \sum_{k=1}^6 w_k = 1 \quad (15)$$

Considering the constraints on decision variables, we believe that there are mainly the following three limitations:

- Water level limit

The optimal water level must be a normal level of water level, so its value is between the highest value and the lowest value of its historical data, that is :

$$h_i^L < h_i(t) < h_i^U \quad i = 1, 2, \dots, n \quad (16)$$

where  $h_i^L, h_i^U$  are the lowest and highest water level of historical data in this  $i$  water area.

- Water flow constraint

Similar to the constraint of water level, we define the flow constraint as :

$$Q^L(i) < Q(t, i) < Q^U(i) \quad i = 1, 2, \dots, n \quad (17)$$

where  $Q^L(i), Q^U(i)$  are the lowest and highest water flow of historical data in this  $i$  water area.

- Water level limit

Due to the influence of the earth 's gravity on the flow direction, in theory, the best water level in the downstream waters should be no higher than the water level in its upstream waters, that is:

$$h_i(t) \geq h_{i+1}(t) \quad i = 1, 2, \dots, n \quad (18)$$

Based on the above discussion of decision variables, multiple objectives, and constraints, we write the optimization model of the task below.

$$\max f(h_1(t), \dots, h_n(t)) = w_1 \xi' + w_2 W_e' + w_3 F_{CD}' + w_4 F_{RB}' + w_5 F_E' + w_6 \Phi'$$

subject to

$$\begin{aligned} h_i^L &< h_i(t) < h_i^U & i=1, 2, \dots, n \\ Q^L(i) &< Q(t, i) < Q^U(i) & i=1, 2, \dots, n \\ h_{i+1}(t) &\leq h_i(t) & i=1, 2, \dots, n \end{aligned} \quad (19)$$

where function  $f(\cdot)$  is an abstract objective function from decision variable  $h_i(t)$  to the optimal value. When  $t$  is determined, the optimal water level of water area  $i$  is the value of the corresponding decision variable  $h_i(t)$  when the optimal value is reached.

## 4.6 Solution and Evaluation of Results

Based on existing Water Level Data, basin area and other data, we calculated an optimal Water Level Interval that can ensure the maximization of optimization function(19) in each period and drew it in Figure 4. The error value is considered, the optimal Water

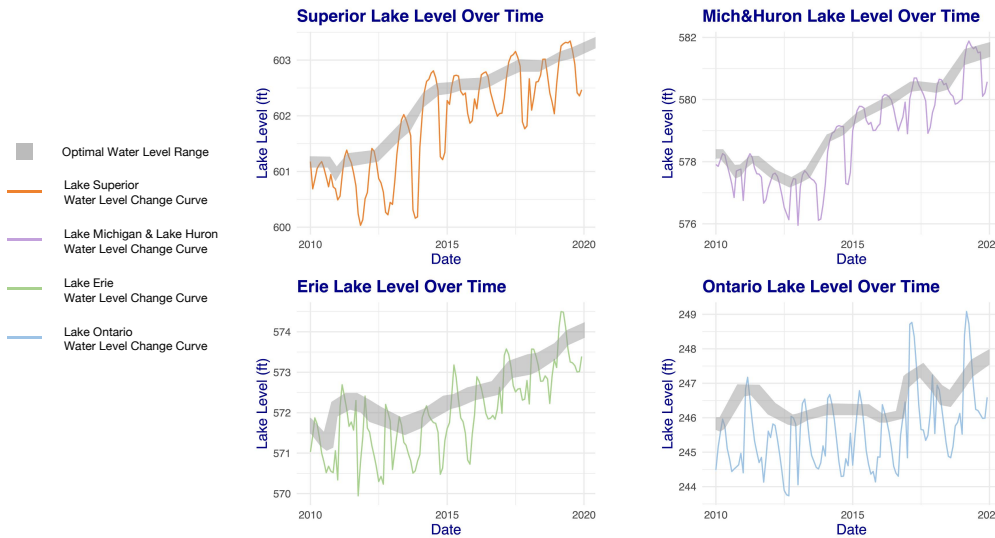


Figure 4: Best Water Level Interval of The Great Lakes

Level Interval is represented in the graph using gray bands, with broken lines representing the Water Level of The Great Lake in a given time frame. It is worth noting that the gray band here is a variable band, which changes over time, which means that we calculated the water level as expected.

## 5 Task 2: The Algorithm to Maintain Optimal Water Levels

### 5.1 Cylinder Equivalent Model of Water Capacity

In order to better study the relationship between the flow changes between the connected waters, we establish the cylinder equivalent volume model of each water area and

Table 4: Data and Database Websites

| Database Names                                         | Database Websites                                                                                             |
|--------------------------------------------------------|---------------------------------------------------------------------------------------------------------------|
| Great Lakes Environmental Research Laboratory[2][1][5] | <a href="https://www.glerl.noaa.gov/data/">https://www.glerl.noaa.gov/data/</a>                               |
| Great Lakes Essential Resources: GIS Data              | <a href="https://research.lib.buffalo.edu/greatlakes/gis">https://research.lib.buffalo.edu/greatlakes/gis</a> |
| Natural Resources Department                           | <a href="https://gisdata.mn.gov/dataset/">https://gisdata.mn.gov/dataset/</a>                                 |
| Great Lakes GIS                                        | <a href="https://www.glc.org/greatlakesgis/">https://www.glc.org/greatlakesgis/</a>                           |
| Canada Energy Regulator                                | <a href="https://www.cer-rec.gc.ca/en/data-analysis/">https://www.cer-rec.gc.ca/en/data-analysis/</a>         |
| Government of Canada                                   | <a href="https://open.canada.ca/data/en/dataset/">https://open.canada.ca/data/en/dataset/</a>                 |
| Tides and Currents                                     | <a href="https://tidesandcurrents.noaa.gov/">https://tidesandcurrents.noaa.gov/</a>                           |

use it for other calculations. The equivalent volume model is shown in the Figure 5.

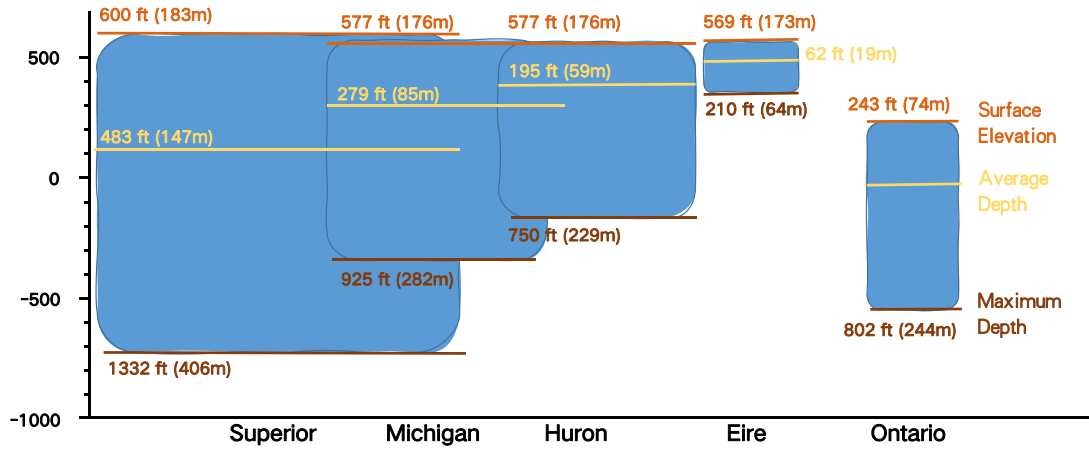


Figure 5: Equivalent volume of each Great Lake

Knowing that the water volume is  $V_i(t)$ , then the water level  $h_i(t)$  is an invariant, we can calculate the cylinder equivalent bottom area  $S_i$  of the bottom area of the irregular water area  $i$ , so that:

$$V_i(t) = S_i h_i(t) \quad (20)$$

Furthermore, the significant advantage of this equivalent form is that the optimal water level problem can be transformed into the optimal capacity problem, that is

$$V_i^*(t) = S_i h_i^*(t) \quad (21)$$

Therefore, given an optimal water level, there is an optimal capacity corresponding to it uniquely. The optimal capacity  $V^*(t)$  obtained from equation (21) is:  $V^*(t) = (V_1^*(t), \dots, V_n^*(t))$ . Therefore, all the following discussions based on the optimal water level can be reduced to the discussion of the optimal volume.

## 5.2 Differential Equation Model of Water Capacity

We rewrite the *Water Balance Equation*(1) from Task 1 in the form of the following differentials.

$$\frac{V_i(t) - V_i(t-1)}{\Delta t} = Q_{in}(t, i) - Q_{out}(t, i) - Q_{quote}(t, i) \quad (22)$$

the equation (22) shows that the water balance equation is a discrete form of the differential equation of the change of the water capacity of the water area  $i$ . And the water balance differential equation for each water area of the Great Lakes system(except the Atlantic) according to the defined longest flow path  $\mathcal{P}$  is:

$$\begin{cases} \frac{dV_1(t)}{dt} = -Q_{out}(t, 1) - Q_{quote}(t, 1) + R(t, 1) \\ \frac{dV}{dt} = \mathbf{Q}_{in} - \mathbf{Q}_{out} - \mathbf{Q}_{quote} + \mathbf{R} \\ \mathbf{V}(0) = \mathbf{V}_0 \end{cases} \quad (23)$$

where

$$\begin{cases} \mathbf{V}(t) = (V_2(t), V_3(t), \dots, V_{n-1}(t))^T \\ \mathbf{V}_0 = (V_1^{(0)}, V_2^{(0)}, \dots, V_{n-1}^{(0)})^T \\ \mathbf{Q}_{in} = (Q_{in}(t, 2), Q_{in}(t, 3), \dots, Q_{in}(t, n-1))^T \\ \mathbf{Q}_{out} = (Q_{out}(t, 2), Q_{out}(t, 3), \dots, Q_{out}(t, n-1))^T \\ \mathbf{Q}_{quote} = (Q_{quote}(t, 2), Q_{quote}(t, 3), \dots, Q_{quote}(t, n-1))^T \\ \mathbf{R} = (R(t, 2), R(t, 3), \dots, R(t, n-1))^T \end{cases} \quad (24)$$

$R(t, i)$  is a perturbation function for each  $i$ . Considering the corresponding actual waters in  $\mathcal{P}$ ,  $i = 1, 9$  correspond to Lake Superior and Lake Ontario respectively, and the two lakes have Sault Ste. Marie and Moses-Saunders Dam respectively. So the  $Q_{out}(t, 1)$  and  $Q_{out}(t, 9)$  are taken in a special way, which we discuss in more detail in the next part.

Next, we discuss the function on the right side of the equation(23).

- $Q_{quote}(t, i)$ :

Set up according to the actual meaning, like actual urban water consumption.

- $Q_{out}(t, i)$ :

**Definition: Natural Flow** The flow of water under natural conditions without the influence of artificial facilities.

**Definition: Artificial Flow** The flow of water in the vicinity of an artificial facility where the flow of water is controlled in accordance with perceived wishes.

1.  $i \neq 1, 9$ :  $Q_{out}(t, i)$  is Natural Flow. Fit historical flow and water level data in Section 4.5.1.
2.  $i = 1, 9$ :  $Q_{out}(t, i)$  is Artificial Flow. Based on the control algorithm in Plan 2014, we improved it by adding factors, and relevant improvements will be discussed below.

## 5.3 New Optimal Water Level Control Algorithm - Plan 2024

### 5.3.1 Review of Plan 2014 Algorithm[3]

Plan 2014 consists of two parts : Rule curves and Lake release control equation.

- Rule curves

Lake releases mainly depend on a sliding rule curve derived from the initial stage-discharge relationship before the project, which is then adapted to current long-term supply conditions. The pre-project relationship from Caldwell and Fay is[4]

$$pr = \beta_0[L - 0.0014(Y - 1985) - 69.474]^{\beta_1} \quad (25)$$

where  $pr$  is the value of pre-project release,  $\beta_0 = 555.823$ ,  $\beta_1 = 1.5$ .

- Lake release control equation

The water flow calculated using this equation is adjusted for recent supply conditions.

$$Q_{out}(i, t) = pr + \mu_{supply} \quad (26)$$

where  $\mu_{supply}$  represents supply conditions.

$$\mu_{supply} = \begin{cases} \mu_{supply}^+ & Q_{in}(t, i) > \overline{Q_{in}(i)} \\ -\mu_{supply}^- & Q_{in}(t, i) < \overline{Q_{in}(i)} \end{cases} \quad (27)$$

where

$$\begin{cases} \mu_{supply}^+ = \left[ \frac{F_{NTS} - A_{NTSavg}}{A_{NTSmax} - A_{NTSavg}} \right] P_1 \cdot C_1 \\ \mu_{supply}^- = \left[ \frac{A_{NTSavg} - F_{NTS}}{A_{NTSavg} - A_{NTSmin}} \right] P_2 \cdot C_1 \end{cases} \quad (28)$$

and  $F_{NTS}$  is a supply index based on the net total supply for the past 52 weeks (48 quarter-months), and  $A_{NTS}$  represents the maximum, minimum and average statistics of the annual net total supply series.  $P_1$ ,  $P_2$  serve to accelerate or decelerate the rate of flow adjustment,  $C_1$  is a constant parameter set by water level optimization( see section 5.3.3 for details ), and  $\overline{Q_{in}(i)}$  denotes the historical contemporaneous mean of the inflow.

### 5.3.2 Data-based Improvement——Flow Factor $\tau$

When Lake Ontario experienced record water levels in 2017 and 2019, its water level control maintenance algorithm, the 2014 plan, caused controversy.

From the Figure 6, we know that:

1. The direct cause of the high water level anomaly is that the water level rises too fast, that is, the water level increment in April is too large.
2. We can further speculate that Plan 2014 did not respond well to the sharp increase in water level increments, which led to the emergence of abnormal water levels. It can also be considered that Lake Ontario did not increase the drainage volume in time to reduce the water level.

In Plan 2014 's approach specification[3], we note about such a flow limit:

J Limit – maximum change in flow from one week (or quarter-month) to the next unless another limit takes precedence.

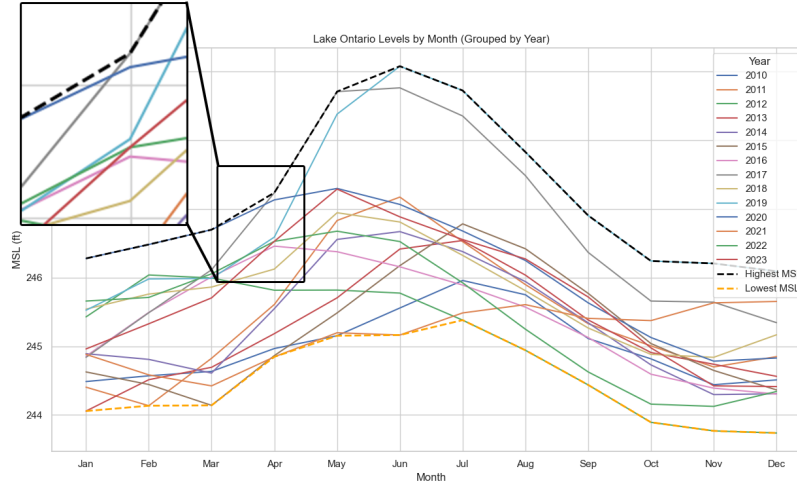


Figure 6: Water level changes in Lake Ontario with Zoom in

J-limit limits the flow increment, so we add a flow-related factor to the equation, which can control the outflow in real time according to the outflow change.

- Level factor  $\tau$

We define the  $\tau$  as:

$$\tau = \begin{cases} \tau^+ & V_i(t) > V_i^*(t) \\ -\tau^- & V_i(t) < V_i^*(t) \end{cases} \quad i = 1, 9 \quad (29)$$

where:

$$\begin{cases} \tau^+ = \left[ \frac{Q_{out}(t-1,i) - \overline{Q}(i)}{Q^U(i) - \overline{Q}(i)} \right] P_3 \cdot C_9 \\ -\tau^- = \left[ \frac{\overline{Q}(i) - Q_{out}(t-1,i)}{\overline{Q}(i) - Q^L(i)} \right] P_4 \cdot C_9 \end{cases} \quad i = 1, 9 \quad (30)$$

the meaning of  $P_3$ ,  $P_4$ ,  $C_9$  is completely similar to that of  $P_1$ ,  $P_2$ ,  $C_1$ . Then we update the lake release control equation(26) in the original Plan 2014, and the new Plan 2024 control equation is in the form of:

$$Q_{out}(t,i) = pr + \mu_{supply} + \tau \quad (31)$$

Compared with the original Plan 2014, the new control algorithm is more effective for the change of water level. It can be seen from equation(29)(30)(31) that when the water level exceeds the optimal value, the factor  $\tau$  makes the outflow gradually increase, and the outflow value will draw on the outflow information at the previous moment, which makes the effect of the factor  $\tau$  to increase the outflow accumulate over time, and then has a stronger control effect on the water level.

### 5.3.3 Determination of the Optimal Parameter Combination

We want to know, in what parameter set  $(C_1, C_9)$  values, the solution is closest to the optimal water volume  $V^*(t)$ , which uses the optimization method. In this optimization, in order to measure the degree of proximity, the following distance  $d$  is defined :

$$d(V, V^*) = \|V^* - V\|_1 \quad (32)$$

where the  $V(t) = (V_1(t), V_2(t), \dots, V_{n-1}(t))$  is the solution of equation(23) obtained by optimisation method in *Algorithm 1* below. The smaller the distance  $d$  is, the better the solution  $V(t)$  is. We call the solution that minimizes the value of  $d$  an *approximate optimal solution*  $\tilde{V}$ , and the pseudo-code for solving equation and parameter optimization are as follows.

---

**Algorithm 1** ODE Solving and Parameter Optimization on L-BFGS-B

---

The algorithm uses the method proposed above to solve the ODE under the parameters, so as to solve the optimal parameters

**Initialize** parameters  $parms \leftarrow \{C_1 = 0.1, C_9 = 0.1\}$

**Define** initial value  $V_{\text{initial}} \leftarrow \{b, V_1, V_2, \dots, V_{n-1}\}$

**Define** time vector  $times \leftarrow \text{seq}(0, Year_{Days}, \text{by} = 1)$

**Define**  $Q_{\text{out}} \leftarrow pr + \mu_{\text{supply}} + \tau$

**for** Each time period  $t \in times$  **do**

    Using  $t$  to form  $Q_{\text{in}}, Q_{\text{out}}, Q_{\text{quote}}$

**for**  $i = 1$  to  $n - 1$  **do**

**if**  $i == 1$  or  $i == 9$  **then**

$i == 1 ? C_1 : C_9$

$dV[i] \leftarrow C \cdot (Q_{\text{in}}[i] - b) - Q_{\text{out}}[i] - Q_{\text{quote}}[i]$

**else**

$dV[i] \leftarrow Q_{\text{in}}[i] - b - Q_{\text{out}}[i] - Q_{\text{quote}}[i]$

**end if**

**end for**

    Update  $V$  through  $dV$

**end for**

Using optimal function to justify  $parms$  to minimize objective function

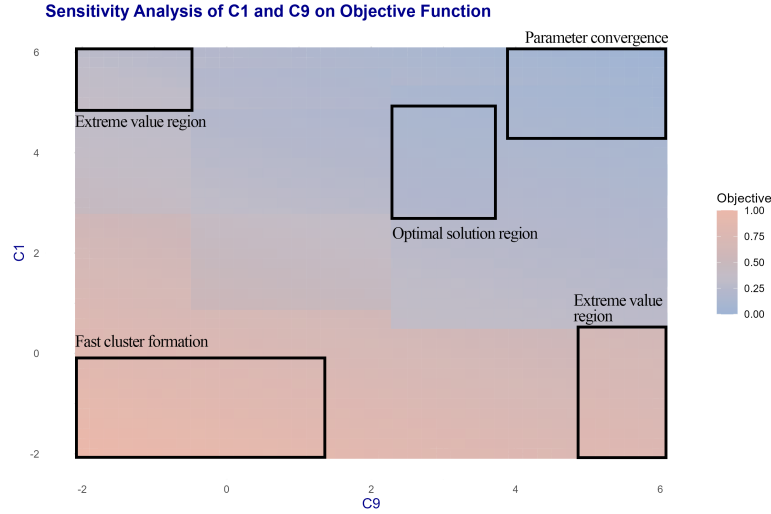
**return** Optimized parameters  $C_1, C_9$  to minimize the objective function and Solutions are the justified lake water volume.

---

With the help of *Algorithm 1*, we draw the objective function values under different parameter sets, as shown in the heat map Figure 7.

In Figure 7, it is not difficult to observe that the optimal solution is attained in the upper-middle part of the heatmap. At the beginning of the optimization, as the initially set values gradually increase, the parameters cause the objective value to converge rapidly. However, when one parameter (either  $C_1$  or  $C_9$ ) becomes excessively large while the other remains small, the objective value becomes extremely skewed, leading to the emergence of an Extreme Value Region. Similarly, when both  $C_1$  and  $C_9$  are significantly large, the parameters drive the objective value to converge to an extreme value, both scenarios being undesirable. Ultimately, we located the best parameter combination within the Optimal Solution Region. Therefore, the parameter set  $(C_1, C_9)$  is **(3.4, 2.7)**.

The figure and analysis also clearly demonstrate that our strategy is resilient to the impact of extreme data (such as the major events that occurred in 2017), with occasional extreme data points not affecting the model's ability to discover the optimal solution globally.

Figure 7: Sensitivity Analysis of  $C_1$  and  $C_9$  on Objective Function

## 6 Task 3: Sensitivity Analysis and Performance of Plan 2024

### 6.1 The Sensitivity of the Control Algorithms for the Outflow

In order to evaluate the sensitivity of our control algorithms for the outflow of the two control dams, we need to find out the direct variable parameters in the algorithm and analyze their effects on the outflow.

Checking the equation (31), the variable parameters we selected are shown in the Table 5, and they are directly related to the value of the  $Q_{out}(t, i)$ .

Table 5: Two groups of variable parameters

| Flow rate adjustment parameters | Water level optimization parameters |
|---------------------------------|-------------------------------------|
| $P_1, P_2, P_3, P_4$            | $(C_1, C_9)$                        |

We quantify the sensitivity of the outflow  $Q_{out}(t, i)$  to parameters : the corresponding change ratio of the  $Q_{out}(t, i)$  when the parameter changes a certain proportion, this can be measured by the ratio of the two, that is, We define the sensitivity coefficient S:

$$S(x, para) = \left| \frac{\Delta x}{\Delta para} \cdot \frac{para}{x} \right| \quad (33)$$

where  $x$  is the detection dependent variable of the parameter.

By constructing the system of ordinary differential equations under Equation 5.3 and solving for parameter optimization in *Algorithm 1*, we obtained the normalized (ratio-standardized method) Objective Function Value and the convergence behavior of each parameter under the advancement of Water flow, as illustrated in Figure 8.

In Figure 8, it is evident that we set all parameter initial values to the same value under normalization. However, during the iterative process of minimizing the objective value (minimizing error), two distinct groups of parameter divergence emerged:  $C_1$  and  $C_9$  gradually converged towards a value slightly to the left or right of the normalized

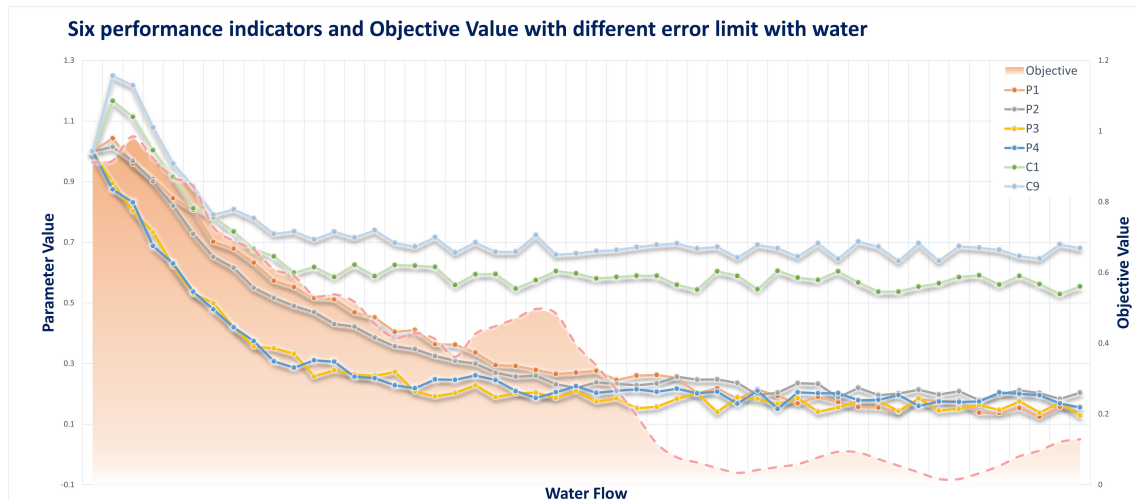


Figure 8: Performance Indicators(Parameters) & Objective Value with Water Difference Error Limit

midpoint, suggesting that our optimal parameter solution is likely to appear around the central region, markedly differing from the results shown in Figure 7.  $P_1$ ,  $P_2$ ,  $P_3$ , and  $P_4$  formed two subgroups, differing only in their rates of convergence. This indicates that the impact experienced at the rear of the watershed is subject to a certain degree of lag, which in turn slows down the convergence rate, but ultimately these four parameters converged to a smaller value that means they are insensitive to the perturbations. This demonstrates that as our iteration progresses, the influence of Equations (29), (30), and (31) gradually diminishes, yet they remain indispensable variables in the outcome.

## 6.2 Taking History as A Mirror——Plan 2024 's Backtesting Performance

We are interested in the comparison of our management and control strategies with previous models. It is worth noting that in 2017, Plan 2014 was questioned in response to the adverse performance of record water levels. So will Plan 2024 have a better performance at that time?

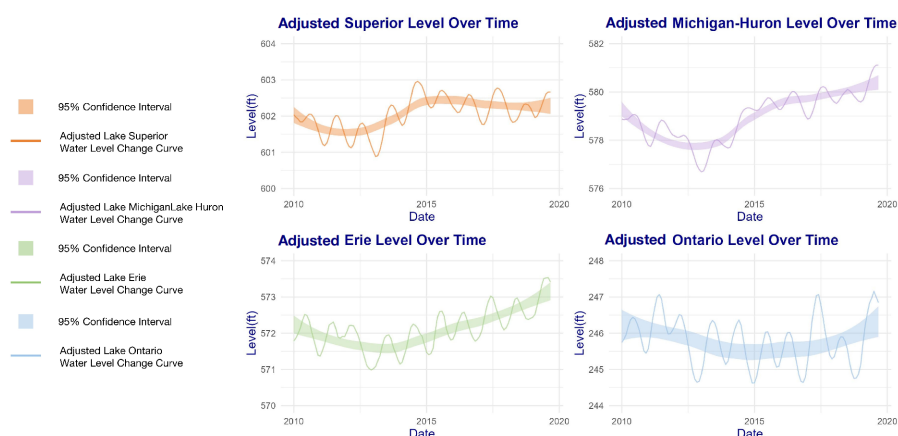


Figure 9: Adjusted Lake Level

In Figure 9, through the application of our control algorithm, we obtained the adjusted optimal water levels along with their 95% confidence intervals. It is evident that the adjusted water levels are closer to the optimal levels, and the fluctuations in water levels are smoother. Notably, the explosive water levels in 2017 and 2019 did not affect the control outcomes.

According to our management and control methods, based on the water level and flow rate at the beginning of 2017, the water volume and water level of the Great Lakes throughout the year are shown in the Figure 10.

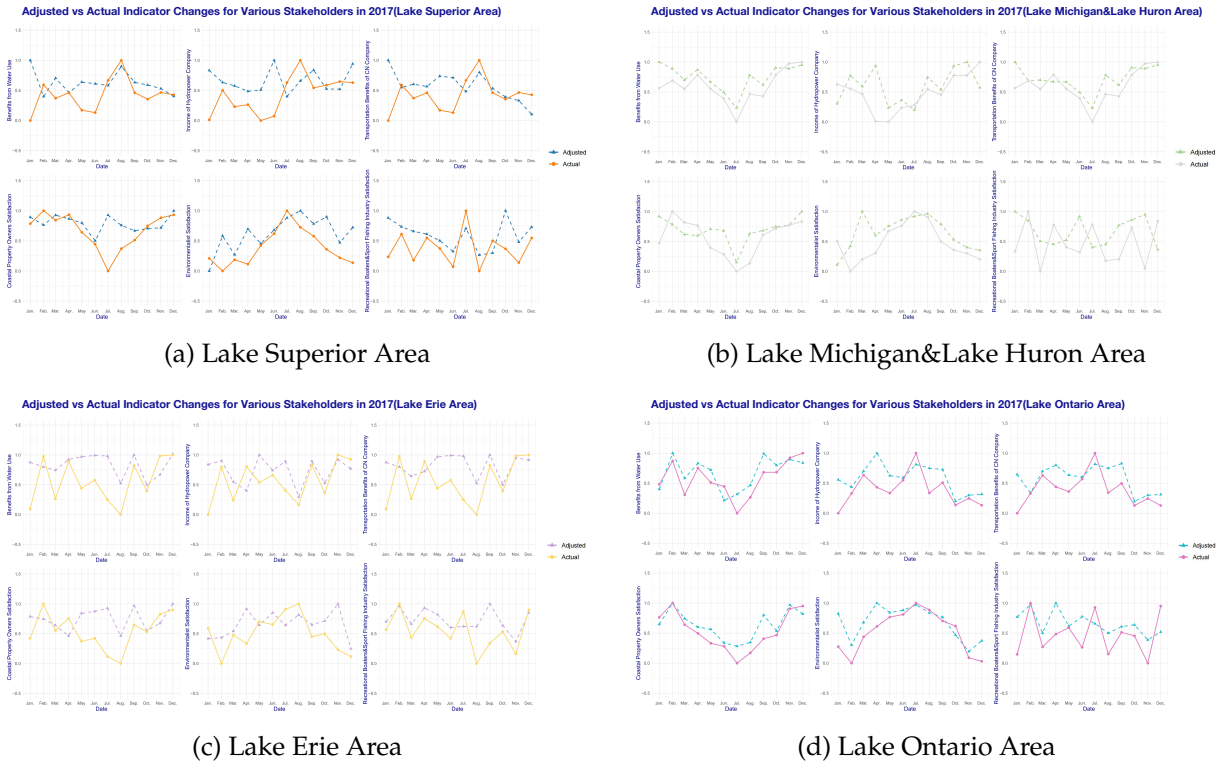


Figure 10: Adjusted vs Actual Indicator Changes for Various Stakeholders in 2017

From the Figure 10, we can see that:

1. For the Great Lakes, after using the new Plan 2024 control algorithm, the performance of various quantitative indicators throughout the year has been significantly improved compared with Plan 2014.
2. In fact, the water level control results of Plan 2024 are not required to be 'perfect'. Some indicators will not perform as well as the Plan 2014 algorithm at some time, but overall, Plan 2024 is better.

Therefore, Plan 2024 passes the backtesting test, which will make all stakeholders more satisfied as a whole.

## 7 Task 4: Sensitivity Analysis about the Changes in Environmental Conditions

The change of environmental conditions will lead to the change of water volume in the water area, which can be attributed to the disturbance term in the differential equation of water volume.

### 1. Precipitation

Precipitation is the equivalent depth of precipitation per unit area of an area per unit time. Therefore, precipitation will affect the water capacity  $V_i(t)$  of area  $i$ , that is, it will cause perturbation to the water capacity. The relationship between the change of precipitation and water capacity in a certain water area  $i$  is,

$$\frac{V_i(t+1) - V_i(t)}{\Delta t} = l_i(t)S_i. \quad (34)$$

### 2. Winter Snowpack

Winter Snowpack is noteworthy. Taking the Ottawa River as an example, generally speaking, the increase and decrease of snowpack will directly affect the water capacity of the St. Lawrence River downstream. The snowpack will cause perturbation to  $V_{10}(t)$ , and the relation is

$$\frac{V_{10}(t+1) - V_{10}(t)}{\Delta t} = v_{melt}(t). \quad (35)$$

In the same way, the mountain snowpack around Lake Superior has a similar effect on its water capacity.

### 3. Ice Jams

This study[6] shows that ice jams cause the water level to rise in the waters where they are located. That is, the ice jam will increase the water capacity of water area  $i$ , and the relationship is as follows,

$$\frac{V_i(t+1) - V_i(t)}{\Delta t} = \Omega(t) \quad (36)$$

where  $\Omega(t)$  is a measure of ice jam severity over time.

In summary, the precipitation term of water  $i$  can be written as

$$R_i(t) = l_i(t)S_i + v_{melt}(t) + \Omega(t) \quad (37)$$

In addition, other environmental conditions, such as water temperature, air temperature, and evaporation rate, etc., can establish a numerical relationship with the above three factors or find a change relationship with the water capacity.

When the above perturbation exists, the solution of the differential equation model (23) changes when the flow rate adjustment parameters  $P_1, P_2, P_3, P_4$  remain unchanged. In order to ensure that the changed solution is still an *approximate optimal solution*, the value of the water level optimization parameter will change correspondingly. Combined with equation (30), it can be seen that Plan 2024 adjusts the outflow according to the disturbance of factors.

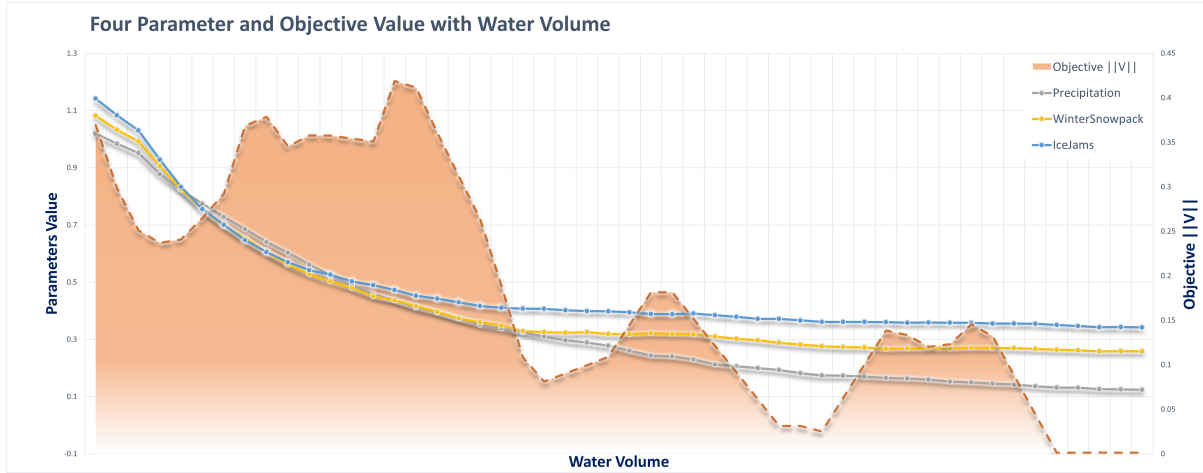


Figure 11: Four Parameter and Objective Value with Different Error Limit with Water Volume

In order to quantify the robustness of this adjustment, we use the sensitivity coefficient to obtain the results in Figure 11.

In Figure 11, by sorting the Water Volume to generate a sequence of data, we then tested the impact of three parameters (Precipitation, Winter Snowpack, Ice Jams—all of which have been normalized) on the Objective  $\|V\|_1$  (the target loss value, with closer to 0 indicating better performance). It can be observed from the figure that, as the sequence progresses, the parameters gradually converge, and the Objective  $\|V\|_1$  similarly decreases towards 0, which is insensitive to the perturbations and precisely the outcome we desired to observe.

## 8 Task 5: New Assessment of Lake Ontario

Lake Ontario's water level management has been under the spotlight since two record levels were recorded in 2017 and 2019.

After that, the stakeholders of the lake may have expectations or preparedness for the possible high water level, and then adjust the demands. But more importantly, under the new management and control strategies in the future, what impact will various stakeholders have on water level management?

In order to study these effects, we established a multiple linear regression (MLR) model :

$$h_9(\xi', W_e', F_{CD}', F_{RB}', F_E', \Phi') = \theta_0 + \theta_1 \xi' + \theta_2 W_e' + \theta_3 F_{CD}' + \theta_4 F_{RB}' + \theta_5 F_E' + \theta_6 \Phi' + \epsilon_t \quad (38)$$

where:

$$\epsilon_t \sim i.i.d.N(0, \sigma_t^2)$$

and  $\theta_0$  is the intercept term,  $\sigma_t^2$  is the square of the fitting residual,  $\theta_1, \dots, \theta_6$  are the regression coefficients of each indicators, they can represent the strength of the influence of the indicator, that is, the magnitude of the change in the water level under the same indicator change. Obviously, the greater the absolute value of the coefficient, the stronger the influence ability.

We use the optimal water level at each time calculated in Task 1 and the time-varying data of each quantitative indicator after standardization, the coefficient results of MLR are shown in the Figure 12.

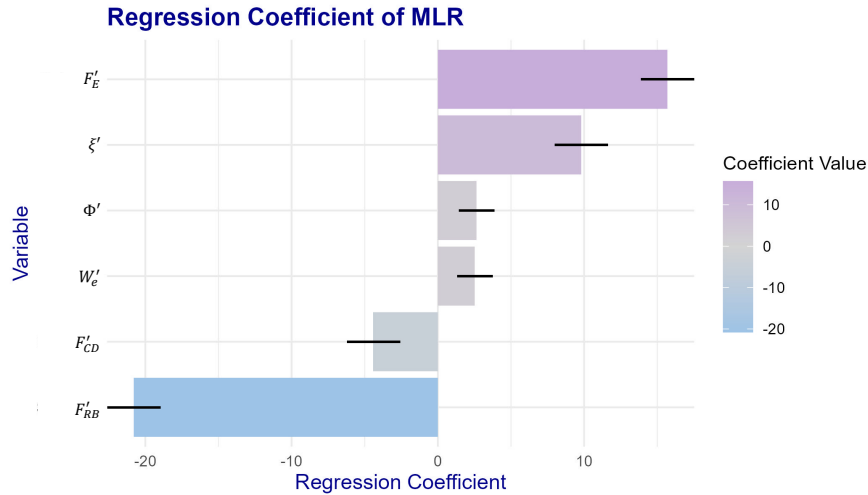


Figure 12: The results of regression coefficients of MLR

From the Figure 12, we know that:

1. If recreational boaters, fishing boats and property owners on Lake Ontario, want to get more benefits, the optimal water level should be lowered, especially for property owners.
2. For environmentalists and shipping companies, in order to get more benefits growth, they will want to greatly improve the optimal water level.
3. For hydropower companies and the companies responsible for municipal and industrial water supply, when their water benefits are increased, the requirements for optimal water levels are small, although the optimal water levels will rise.

## 9 Evaluation of the model

### 9.1 Strengths

- The network structure of the network model of the Great Lakes region is based on the water structure and the direction of the water flow. Therefore, any water flow with head, tail and direction in the water network can be modeled as a path in the network model.
- When using multi-objective optimization to determine the optimal water level, the demands of various stakeholders are fully considered. According to the variables required by each stakeholder, many practical quantitative indicators are established.
- The new control algorithm Plan 2024 inherits the advantages of Plan 2014 in algorithm design. Considering the flow limits principle of Plan 2014, a new factor  $\tau$  is

added, which has stronger ability to capture real-time flow changes and stronger ability to control water level.

## 9.2 Weaknesses

- In the basic assumption, the assumption that the velocity of the cross section of the water is the same at any time is too strong. In order to extend the model to practical use, it is also necessary to calculate the total flow of the cross section by means of calculus.
- In the sensitivity analysis, no specific quantitative indicators have been established for various other indicators, such as evaporation rate and water temperature. This work will be further studied in the future.

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## Memo

**To:** The dear International Joint Commission Leadership

**From:** Team 2413321

**Date:** February 6th, 2024

**Subject:** A better choice than Plan 2014: Plan 2024

In the memorandum, we would like to introduce to you a more excellent management and control strategy for the Great Lakes waters : the optimal water level control algorithm based on optimizing the demands of various stakeholders-Plan 2024. In fact, Plan 2024 is a significant improvement based on the design idea of Plan 2014, and it has effectively countered the controversy caused by the performance failure of Plan 2014 in 2017 and 2019.

First, the key features of Plan 2024. In the first part, in order to optimize the interest needs of various stakeholders, the strategy includes the quantitative evaluation indicators of six major stakeholders, and combines the multi-objective optimization method to calculate the time-varying optimal water level of each water area. In the second part, in order to solve the shortcomings of the original Plan 2014 algorithm in dealing with sudden flow changes, based on its design, the flow factor  $\tau$  is introduced to capture the dynamic changes of real-time water flow to adjust the outflow, and the algorithm parameters are flexibly adjusted according to the time-varying optimal water level. These two key features are the soul of Plan 2024.

Secondly, the setting of parameters in the model. The key parameter in the model is the optimization parameter  $(C_1, C_9)$  in the flow factor tau. Using the data of water capacity and water level at the initial time of each water area, the water level at a later time is obtained by means of the differential equation of flow, and then the optimization method is used to calculate the parameter value when the water level is closest to the optimal water level.

Lastly, Based on the model comparison of the backtesting method, we use the water level and water capacity in early 2017 to observe whether Plan 2024 performs better than Plan 2014. The results show that under the action of Plan 2024, all the stakeholders of the Great Lakes have different degrees of improvement, which also proves that Plan 2024 performs better.

We sincerely hope that you will consider our management and control strategies - Plan 2024